SE-6221/100

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B. E. - II (Sem. - III) (IC) Examination

May / June – 2011

Engineering Maths - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. - II (Sem - III) (IC)**

Name of the Subject :
☛ **Engineering Maths - III**

☛ Subject Code No. : **6 2 2 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figure on right indicate marks.

1 (a) Do as directed : **10**

(i) Evaluate $\int_{-1}^{3}\int_{x^2}^{x+2} dydx$.

(ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that div $\vec{r} = 3$ and curl $\vec{r} = \vec{0}$.

(iii) State Gauss-Divergence theorem for a vector point function.

(iv) Show that the vector

$\vec{F} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$ is solenoidal.

(v) Find that Jacobian of the transformation from cartesian coordinates to polar coordinates.

(b) Attempt any **three** : **12**

(i) Evaluate $\int_{0}^{a}\int_{0}^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$ by changing into polar coordinates.

(ii) Change the order of integration of the integral

$\int_{0}^{a}\int_{0}^{\sqrt{a^2-x^2}} f(x,y)dydx$.

(iii) Calculate the volume of the solid bounded by the surface $x=0$, $y=0$, $x+y+z=1$ and $z=0$.

(iv) Find the mass of the plate which is inside the circle $r=2a\cos\theta$ and outside the circle $r=a$, if the density is $\rho=kr$.

2 (a) Attempt any **two** : **6**

(i) Define curl of a vector function. Show that the field given by $\vec{F}=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)\hat{j}$ is irrotational.

(ii) Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point (5,0,4).

(iii) A vector field is given by $\vec{F}=\sin y\hat{i}+x(1+\cos y)\hat{j}$. Evaluate the line integral over the circular path given by $x^2+y^2=a^2$, $z=0$.

(b) Attempt any **two** : **8**

(i) Verify divergence theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by the cylinder $x^2+y^2=4$, $z=0$, $z=3$.

(ii) Verify Green's theorem in the plane for $\oint_C \left(2xy-x^2\right)dx+\left(x^2+y^2\right)dy$ where C is the boundary of the region enclosed by $y=x^2$ and $x=y^2$.

(iii) Verify Stoke's theorem for the vector field $\vec{F}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ over the upper half surface of $x^2+y^2+z^2=1$ bounded by its projection on the xy-plane.

3 (a) Express $f(x)=x^2$ as a half-range cosine series in $0<x<\pi$. **4**

(b) Attempt any **two** : **10**

(i) Obtain the Fourier series to represent

$$f(x)=\frac{1}{4}(\pi-x)^2,\quad 0<x<2\pi.$$

(ii) If $f(x)=0$, $-\pi\le x\le 0$

$=\sin x$, $0\le x\le 0$

find the Fourier series to represent $f(x)$, Hence show that $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots\ldots=\frac{1}{2}$.

(iii) Expand $f(x)=e^{-x}$ as a Fourier series in the interval $(-l,l)$.

4 (a) Do as directed : **10**

(i) State relation between Beta and Gamma functions. Find the value of $\beta(2,6)$.

(ii) Show that error function is an odd function.

(iii) Write two dimensional Laplace equation.

(iv) Define Inverse Laplace transform and hence find $L^{-1}\left\{\frac{1}{S-5}\right\}$.

(v) Define analytic function and state necessary and sufficient conditions for $f(z)$ to be analytic.

(b) Attempt any **two** : **6**

(i) Evaluate $\int_0^\infty \frac{x^4}{\left(1+x^2\right)^4}dx$.

(ii) Show that $\int_0^\infty x^{2n-1}e^{-ax^2}\,dx=\frac{\Gamma(n)}{2a^n}$.

(iii) Prove that $\int_0^\infty e^{-x^2-2ax}\,dx=\frac{\sqrt{\pi}}{2}e^{a^2}\left(1-erf(a)\right)$

(c) Solve any **two** : **6**

(i) $(mz-ny)p+(nx-lz)q=ly-mx$

(ii) $x(y-z)p+y(z-x)q=z(x-y)$

(iii) $\left(y^2+z^2\right)p+xyq=xz$

5 (a) Attempt any **one** : **6**

(i) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y=a\sin\frac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t)=a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$.

(ii) Solve the following problem :

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < l$$

$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(l,t) = 0$ for all t and $u(x,0) = x$.

(b) Attempt any **two** : 8

(i) Find the Laplace transform of $f(t) = t^3 e^{-3t}$.

(ii) Find the inverse Laplace transform of $\frac{S+1}{S^2+2s}$.

(iii) Solve by method of transforms

$$x'' - 3x' + 2x = 1 - e^{2t}, \quad x(0) = 1$$
$$x'(0) = 0.$$

6 (a) Attempt any **two** : 8

(i) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express u+iv as an analytic function of z.

(ii) Under the transformation $w = \frac{1}{z}$, find the image of $|z - 2i| = 2$.

(iii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$.

(b) Attempt any **two** : 6

(i) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$.

(ii) Evaluate $\oint_c \frac{e^z}{(z-1)(z-4)} dz$, where c is the circle $|z| = 2$.

(iii) Evaluate $\oint_c \frac{\cos z}{(z-\pi)^3} dz$, where c is the circle $|z-1| = 3$.